Matrix Mutliplication References

Articles and book chapters on matrix multiplication.

Some documents and comments should only be used internally. Confidential documents are marked as such. Please take note of legal notices. In case something is missing, please notify Axel Kemper

Note 1: There is a "find" button at the end of the page. It allows you to list all the references which have a keyword in common.

Note 2: Click the small flag at the bottom to see this page in German.

Matrix Mutliplication and SAT References
Articles and book chapters on matrix multiplication and Boolean satisfiability. Some documents and comments should only be used internally. Confidential documents are marked as such. Please take note of legal notices. In case something is missing, please notify Axel Kemper

Note 1: There is a "find" button at the end of the page. It allows you to list all the references which have a keyword in common.

Note 2: Click the small flag at the bottom to see this page in German.

Kemper, Axel: **Verfahren von Makarov zur Multiplikation von 5x5 Matrizen** 01-Jan-2015
(19 pages).

Pan, Victor Y. [City University of New York]: **Matrix Multiplication, Trilinear Decompositions, APA Algorithms, and Summation** 18-Dec-2014
Matrix multiplication (hereafter we use the acronym MM) is among the most fundamental operations of modern computations. The efficiency of its performance depends on various factors, in particular vectorization, data movement and arithmetic complexity of the computations, but here we focus just on the study of the arithmetic cost and the impact of this study on other areas of modern computing. In the early 1970s it was expected that the straightforward cubic time algorithm for MM will soon be accelerated to enable MM in nearly quadratic arithmetic time, with some far fetched implications. While pursuing this goal the mainstream research had its focus on the decrease of the classical exponent 3 of the complexity of MM towards its lower bound 2, disregarding the growth of the input size required to support this decrease. Eventually, surprising combinations of novel ideas and sophisticated techniques enabled the decrease of the exponent to its benchmark value of about 2.38, but the supporting MM algorithms improved the straightforward one only for the inputs of immense sizes. Meanwhile, the communication complexity, rather than the arithmetic complexity, has become the bottleneck of computations in linear algebra. This development may seem to undermine the value of the past and future research aimed at the decrease of the arithmetic cost of MM, but we feel that the study should be reassessed rather than closed and forgotten. We review the old and new work in this area in the present day context, recall some major techniques introduced in the study of MM, discuss their impact on the modern theory and practice of computations for MM and beyond MM, and link one of these techniques to some simple algorithms for inner product and summation. (17 pages).

Ballard, Grey [Sandia National Laboratories]: **Communication-Avoiding Algorithms and Fast Matrix Multiplication** 29-Sep-2014
(78 slides).

Benson, Austin; Ballard, Grey: **A Framework for Practical fast Matrix Multiplication** 26-Sep-2014
(32 slides).

Benson, Austin R.; Ballard, Grey: **A Framework for Practical Parallel Fast Matrix Multiplication** 09-Sep-2014
(14 pages).

Bläser, Markus [Saarland University]: **Fast matrix multiplication and related problems** 09-Jul-2014
(36 slides).

Williams, Virginia Vassilevska [Stanford University]: **Multiplying matrices in O(n^2.373) time** 01-Jul-2014
(73 pages).

Boyer, Brice; Dumas, Jean-Guillaume: **Boyer-Dumas - 2014 - Matrix multiplication over word-size prime Fields using Bini?s approximate formula** 06-May-2014
(14 pages).

Ottaviani, Giorgio [Università di Firenze]: **Complexity of Matrix Multiplication and Tensor Rank** 27-Feb-2014
(37 slides).

Kemper, Axel: **Verfahren von Makarov zur Multiplikation von 5x5 Matrizen** 06-Jan-2014
(17 pages).

Laitinen, Tero; Junttila, Tommi; Niemelä, Ilkka [Aalto University]: **Simulating Parity Reasoning (extended version)** 18-Nov-2013
Propositional satisfiability (SAT) solvers, which typically operate using conjunctive normal form (CNF), have been successfully applied in many domains. However, in some application areas such as circuit verification, bounded model checking, and logical cryptanalysis, instances can have many parity (xor) constraints which may not be handled efficiently if translated to CNF. Thus, extensions to the CNF-driven search with various parity reasoning engines ranging from equivalence reasoning to incremental Gaussian elimination have been proposed. This paper studies how stronger parity reasoning techniques in the DPLL(XOR) framework can be simulated by simpler systems: resolution, unit propagation, and parity explanations. Such simulations are interesting, for exam-ple, for developing the next generation SAT solvers capable of handling parity constraints efficiently. (37 pages).

Landsberg, J.M.: **New Lower Bounds for the Rank of Matrix Multiplication** 29-Oct-2013
The rank of the matrix multiplication operator for nxn matrices is one of the most studied quantities in algebraic complexity theory. I prove that the rank is at least n^2-o(n^2). More precisely, for any integer p\leq n -1, the rank is at least (3-1/(p+1))n^2-(1+2p\binom{2p} {p-1})n. The previous lower bound, due to Blaser, was 5n^2/2-3n (the case p=1). The new bounds improve Blaser's bound for all n>84. I also prove lower bounds for rectangular matrices significantly better than the the previous bound. (6 pages).

Landsberg, J.M.; Ottaviani, Giorgio: **New Lower Bounds for the Border Rank of Matrix Multiplication** 02-Jun-2013
The border rank of the matrix multiplication operator for n × n matrices is a standard measure of its complexity. Using techniques from algebraic geometry and representation theory, we show the border rank is at least $2n^2 - n$. Our bounds are better than the previous lower bound (due to Lickteig in 1985) of $3/2n^2 + n/2 - 1$ for all n >= 3. The bounds are obtained by finding new equations that bilinear maps of small border rank must satisfy, i.e., new equations for secant varieties of triple Segre products, that matrix multiplication fails to satisfy. (11 pages).

Oh, Jinsoo; Kim, Jin; Moon, Byung-Ro: **On the inequivalence of bilinear algorithms for 3 × 3 matrix multiplication** 14-May-2013
Since Laderman showed an algorithm for 3 × 3 matrix multiplication using 23 scalar multiplications, Johnson and McLoughlin used a numerical optimization and human controlled method to give two parameterized algorithms in which the coefficients are rational numbers. The algorithms are inequivalent to Laderman?s one with respect to the transformation introduced by de Groote. We present a simple and fast numerical heuristic for finding valid algorithms. Then we show that many of the obtained algorithms are inequivalent to the published ones. (6 pages).

Hart, Sarah; Hedtke, Ivo; Müller-Hannemann, Matthias; Murthy, Sandeep: **A Fast Search Algorithm for [m, m, m] Triple Product Property Triples Aand An Application for 5 × 5 Matrix Multiplication** 01-May-2013
We present a new fast search algorithm for

Smirnov, A. V.: **The Bilinear Complexity and Practical Algorithms for Matrix Multiplication** 19-Mar-2013
A method for deriving bilinear algorithms for matrix multiplication is proposed. New estimates for the bilinear complexity of a number of problems of the exact and approximate multiplication of rectangular matrices are obtained. In particular, the estimate for the boundary rank of multiplying 3 × 3 matrices is improved and a practical algorithm for the exact multiplication of square n × n matrices is proposed. The asymptotic arithmetic complexity of this algorithm is O(n^2.7743). (15 pages). Russion version: http://www.mathnet.ru/php/archive.phtml?wshow=paper&jrnid=zvmmf&&option_lang=eng

Hedtke, Ivo [Martin-Luther-University Halle-Wittenberg]: **Hedtke - 2013 - (Group-theoretic) Fast Matrix Multiplication** 11-Mar-2013
(25 slides).

Bläser, Markus [Saarland University]: **Fast matrix multiplication** 06-Mar-2013
(59 pages).

Bläser, Markus [Universität des Saarlandes]: **Fast matrix multiplication** 06-Mar-2013
(59 pages).

Murat Cenk, M. Anwar Hasan [University of Waterloo]: **On the Arithmetic Complexity of Strassen-Like Matrix Multiplications** 24-Feb-2013
(20 pages).

Courtois, Nicolas T. [University College London]: **Multiplicative Complexity** 28-Jan-2013
(105 slides).

Filmus, Yuval: **Matrix Multiplication 2** 14-Jan-2013
(14 pages).

Massri, César [University of Buenos Aires]: **Solving a sparse systems using linear algebra** 15-Nov-2012
We generalize a method to compute the solutions of a system of polynomial equations with finitely many solutions. We solve this problem without the need of Gröbner bases. (15 pages).

Petersen, Kaare Brandt; Pedersen, Michael Syskind: **The Matrix Cookbook** 15-Nov-2012
(72 pages).

Ottaviani, Giorgio [Università di Firenze]: **Tensor decomposition and tensor rank from the point of view of Classical Algebraic Geometry** 26-Sep-2012
(57 pages).

Cohn, Henry; Umans, Christopher: **Fast matrix multiplication using coherent configurations** 27-Jul-2012
(18 pages).

Joó, A.; Ekárt, A.; Neirotti, J. P. [Aston University, Birmingham]: **Genetic algorithms for discovery of matrix multiplication methods** 07-Jul-2012
We present a parallel genetic algorithm for nding matrix multiplication algorithms. For 3 × 3 matrices our genetic algorithm successfully discovered algorithms requiring 23 multiplications, which are equivalent to the currently best known human-developed algorithms. We also studied the cases with less multiplications and evaluated the suitability of the methods discovered. Although our evolutionary method did not reach the theoretical lower bound it led to an approximate solution for 22 multiplications. (7 pages).

Gall, Francois Le: **Faster Algorithms for Rectangular Matrix Multiplication** 09-Apr-2012
(37 pages).

N.N.: **Note on the Strassen formulas** 07-Apr-2012
(2 pages).

T.Courtois, Nicolas; Hulme, Daniel; Mourouzis, Theodosis [University College London]: **Multiplicative Complexity And Solving Generalized Brent Equations With SAT Solvers** 31-Mar-2012
(6 pages).

Courtois, Nicolas T.; Hulme, Daniel; Mourouzis, Theodosis [University College London; NP-Complete Ltd.]: **Multiplicative Complexity Gate Complexity Cryptography and Cryptanalysis** 18-Mar-2012
(75 slides).

Filmus, Yuval: **Matrix Multiplication 2** 09-Feb-2012
(10 pages).

Filmus, Yuval: **Matrix Multiplication 1** 02-Feb-2012
(13 pages).

Lipton, R.J.: **A Brief History Of Matrix Product** 01-Feb-2012
(5 pages).

Holtz, Olga [University of California at Berkeley]: **Fast and stable matrix multiplication** 31-Dec-2011
(44 slides).

Williams, Virginia Vassilevska [University of California, Berkeley]: **Breaking the Coppersmith-Winograd barrier** 30-Nov-2011
We develop new tools for analyzing matrix multiplication constructions similar to the Coppersmith-Winograd construction, and obtain a new improved bound on omega < 2.3727. (72 pages).

Courtois, Nicolas T.; Bard, Gregory V.; Hulme, Daniel: **A New General-Purpose Method to Multiply 3x3 Matrices Using Only 23 Multiplications** 19-Aug-2011
(10 pages).

Hedtke, Ivo: **Methods of Matrix Multiplication: An Overviw of several methods and their implementation** 08-Jun-2011
In this overview article we present several methods for multiplying matrices and the implementation of these methods in C. Also a little test program is given to compare their running time and the numerical stability. The methods are: naive method, naive method working on arrays, naive method with the KAHAN trick, three methods with loop unrolling, winograd method and the scaled variant, original STRASSEN method and the STRASSEN-WINOGRAD variant. (25 pages).

Hedtke, Ivo [Jena University]: **Darstellungstheoretische Ansätze in der schnellen Matrixmultiplikation** 13-May-2011
Diploma thesis (58 pages). Slides: http://www.ihmm.de/documents/grueppchen2012.pdf

Hedtke, Ivo: **A Note on the Group-Theoretic Approach to Fast Matrix Multiplication** 11-May-2011
(5 pages).

Hedtke, Ivo; Murthy, Sandeep: **Search and Test Algorithms for Triple Product Property Triples** 11-May-2011
In 2003 COHN and UMANS introduced a group-theoretic approach to fast matrix multiplication. This involves finding large subsets of a group G satisfying the Triple Product Property (TPP) as a means to bound the exponent ? of matrix multiplication. We present two new characterizations of the TPP, which are useful for theoretical considerations and for TPP test algorithms. With this we describe all known TPP tests and implement them in GAP algorithms. We also compare their runtime. Furthermore we show that the search for subgroup TPP triples of nontrivial size in a nonabelian group can be restricted to the set of all nonnormal subgroups of that group. Finally we describe brute-force search algorithms for maximal subgroup and subset TPP triples. In addition we present the results of the subset brute-force search for all groups of order less than 25 and selected results of the subgroup brute-force search for 2-groups, SL(n,q) and PSL(2,q). (14 pages).

Drevet, Charles-Éric; Islam, Md. Nazrul; Schost, Éric: **Optimization Techniques for Small Matrix Multiplication** 06-Oct-2010
(1 Poster).

Bodrato, Marco: **Bodrato - 2010 - Strassen-like Matrix Multiplication for Squares** 30-Jul-2010
(8 pages).

Bodrato, Marco: **Strassen-like-Matrix Multiplication suited for Squaring and Higher Power Computation** 28-Jul-2010
(28 slides).

Drevet, Charles-Éric; Islam, Md. Nazrul; Schost, Eric [Universiyt of Western Ontario]: **Optimization Techniques for Small Matrix Multiplication** 14-May-2010
(32 pages).

Smith, Warren D.: **Fast matrix algorithms and multiplication formulae** 18-Apr-2010
Contains a table of 25 approximate matrix multiplication algorithms, many of them better than the Strassen algorithms (example: 5x5x5 with 90 products) (17 pages).

Pospelov, Alexey [Saarland University]: **Bounds for Bilinear Complexity of Noncommutative Group Algebras** 24-Mar-2010
We study the complexity of multiplication in noncommutative group algebras which is closely related to the complexity of matrix multiplication. We characterize such semisimple group algebras of the minimal bilinear complexity and show nontrivial lower bounds for the rest of the group algebras. These lower bounds are built on the top of Bläser's results for semisimple algebras and algebras with large radical and the lower bound for arbitrary associative algebras due to Alder and Strassen. We also show subquadratic upper bounds for all group algebras turning into "almost linear" provided the exponent of matrix multiplication equals 2. (20 pages).

Islam, Md. Nazrul [University of Western Ontario, Canada]: **A Generalization of Waksman's Matrix Multiplication** 12-Feb-2010
(3 pages).

Islam, Md. Nazrul: **Formulas for Hopcroft-Kerr's Matrix Multiplication Algorithm** 12-Feb-2010
(3 pages).

Srimani, P.K.; Rani, G. Vakula: **Srimani-Rani -2010 - A Comparative Study of Fast Matrix Multiplication Methods (FMMM) by using Vedic Algorithms** 10-Feb-2010
(6 pages).

Anderson, Matthew; Barman, Siddharth: **The Coppersmith-Winograd Matrix Multiplication Algorithm** 06-Dec-2009
(11 pages).

Oh, Seunghyun; Moon, Byung-Ro: **Strassen's Algorithm Revisited by Genetic Search** 02-Dec-2009
(6 pages).

Loos, Sarah M.; Wise, David S.: **Strassen's Matrix Multiplication Relabeled** 01-Dec-2009
A very simple recasting of this classic 7-multiplication recursion improves its time performance for rectangular matrices of order n when n is not a power of two. No time is lost in the rare cases when n = 2^p. A similar relabeling applies, as well, to Winograd's 15-addition variant, for which experiments on n × 2n × n products show an average 25time improvement. (7 pages).

Williams, Virginia Vassilevska; Williams, Ryan [Princeton University]: **Triangle Detection Versus Matrix Multiplication** 05-Nov-2009
(18 pages).

Bläser, Markus [Saarland University]: **Complexity of Bilinear Problems - Computational Complexity** 07-Oct-2009
(46 pages).

Holtz, Olga; Shomron, Noam: **Computational Complexity and Numerical Stability of Linear Problems** 12-Sep-2009
We survey classical and recent developments in numerical linear algebra, focusing on two issues: computational complexity, or arithmetic costs, and numerical stability, or performance under roundoff error. We present a brief account of the algebraic complexity theory as well as the general error analysis for matrix multiplication and related problems. We emphasize the central role played by the matrix multiplication problem and discuss historical and modern approaches to its solution. (16 pages).

Islam, Md. Nazrul [University of Western Ontario]: **Optimization Techniques For Matrix Multiplication** 27-Aug-2009
(95 pages).

Eve, James [Newcastle University]: **On $O(n^2 \log n)$ algorithms for n×n matrix operations** 01-Aug-2009
(42 pages).

Färnqvist, Tommy [Linköping University]: **Lower Bounds for Matrix Multiplication** 09-Jun-2009
Master's Thesis Proposal (2 pages).

Orem, Hendrik [Harvey Mudd College]: **Fast Matrix Multiplication via Group Actions** 01-May-2009
Recent work has shown that fast matrix multiplication algorithms can be constructed by embedding the two input matrices into a group algebra, ap-plying a generalized discrete Fourier transform, and performing the multiplication in the Fourier basis. Developing an embedding that yields a matrix multiplication algorithm with running time faster than naive matrix multiplication leads to interesting combinatorial problems in group theory. The crux of such an embedding, after a group G has been chosen, lies in finding a triple of subsets of G that satisfy a certain algebraic relation. I show how the process of finding such subsets can in some cases be greatly simplified by considering the action of the group G on an appropriate set X. In particular, I focus on groups acting on regularly branching trees. (41 pages).

Bowen, Richard Strong; Chen, Bob; Orem, Hendrik; Schaardenburg, Martijn van [Harvey Mudd College]: **Group-Theoretic Partial Matrix Multiplication** 13-Feb-2009
(14 pages).

Mayr, Ernst W. [Technical University Munich]: **Matrixmultiplikation à la Strassen** 12-Jan-2009
(9 slides).

Bard, Gregory V. [Fordham University]: **New Practical Strassen-like Approximate Matrix-Multiplication Algorithms found via solving a System of Cubic Equations** 15-Dec-2008
(17 pages).

Petersen, Kaare Brandt; Pedersen, Michael Syskind: **The Matrix Cookbook** 14-Nov-2008
(71 pages).

Bard, Gregory V.: **A Practical Algorithm for Massively-Parallel Dense Matrix Multiplication in time n 2.777** 29-Oct-2008
(72 slides).

D'Alberto, Paolo; Nicolau, Alexandru: **Adaptive Winograd's Matrix Multiplications** 01-Jul-2008
(21 pages).

Kleitman, Daniel; Shor, Peter: **Principles of Applied Mathematics: Strassen's Fast Multiplication of Matrices Algorithm and Spreadsheet Matrix Multiplications** 09-May-2008
(11 pages).

Srivastava, Piyush: **Approximative techniques for embedding Matrix Multiplication in Group Algebras** 17-Apr-2008
In this article, we discuss a scheme of embedding matrix multiplication into a group algebra based on the concept of degeneration, and present conjectures which if true would lead to non-trivial estimates of the exponent of matrix multiplication using these schemes. (10 pages).

Bard, Gregory V. [Fordham University]: **On the Rapid Solution of Systems of Polynomial Equations over Low-Degree Extension Fields of GF(2), via SAT-Solvers.** 13-Mar-2008
(25 pages).

Landsberg, J.M.: **Geometry and the Complexity of Matrix Multiplication** 07-Jan-2008
(38 pages).

Gates, Ann Q.; Kreiniovich, Vladik [University of Texas at El Paso]: **Strassen's Algorithm Made (Somewhat) More Natural - A Pedagogical Remark** 07-Nov-2007
(4 pages).

Bini, Dario A. [Pisa University]: **The role of tensor rank in the complexity analysis of bilinear forms** 16-Jul-2007
(46 slides).

Landsberg, J.M.: **Geometry and the Complexity of Matrix Multiplication** 12-Mar-2007
We survey results in algebraic complexity theory, focusing on matrix multiplication. Our goals are (i.) to show how open questions in algebraic complexity theory are naturally posed as questions in geometry and representation theory, (ii.) to motivate researchers to work on these questions, and (iii.) to point out relations with more general problems in geometry. The key geometric objects for our study are the secant varieties of Segre varieties. We explain how these varieties are also useful for algebraic statistics, the study of phylogenetic invariants, and quantum computing. (34 pages).

Bartsch, Benjamin A.: **Laderman's algorithm for 3x3 matrix multiplication** 01-Jan-2007
Newsgroup log (7 pages).

Demmel, James; Dumitriu, Ioana; Holtz, Olga; Kleinberg, Robert [University of California at Berkeley]: **Fast matrix multiplication is stable** 07-Dec-2006
(19 pages).

Bard, Gregory V. [University of Maryland]: **Achieving a log(n) Speed Up for Boolean Matrix Operations and Calculating the Complexity of the Dense Linear Algebra step of Algebraic Stream Cipher Attacks and of Integer Factorization Methods** 05-May-2006
(20 pages).

Landsberg, J.M.: **The Border Rank of the Multiplication of 2 x2 Matrices is seven** 05-Apr-2006
(16 pages).

Brickell, Justin: **Practical Fast Matrix Multiplication** 25-Dec-2005
Project report (15 pages).

Bard, Gregory V.: **Algorithms for Fast Matrix Operations** 07-Dec-2005
(13 pages).

Cohn, Henry; Kleinberg, Robert; Szegedy, Balázs; Umans, Christopher: **Group-theoretic Algorithms for Matrix Multiplication** 10-Sep-2005
(12 pages).

Lang, Stefan [Munich University]: **Matrixalgebra mit einer Einführung in lineare Modelle** 14-Apr-2005
(213 pages). In German

Wayne, Kevin [Pearson-Addison Wesley]: **How to Multiply** 01-Jan-2005
(16 slides).

Lang, Stefan [Munich University]: **Matrixalgebra** 25-Aug-2004
(213 pages).

Yuster, Raphael; Zwick, Uri: **Fast sparse matrix multiplication** 21-Jun-2004
(11 pages).

Kaporin, Igor [Russian Academy of Sciences, Moscow]: **The aggregation and cancellation techniques as a practical tool for faster matrix multiplication** 01-May-2004
(42 pages).

Kakaradov, Boyko [Stanford University]: **Ultra-Fast Matrix Multiplication - An Empirical Analysis of Highly Optimized Vector Algorithms** 01-Apr-2004
(4 pages).

Widgerson, Avi [Princeton University]: **Arithmetic Complexity - A Survey** 23-Dec-2003
(13 pages).

Kleitman, Daniel; Shor, Peter: **Principles of Applied Mathematics: Strassen's Fast Multiplication of Matrices Algorithm and Spreadsheet Matrix Multiplications** 23-Dec-2003
(9 pages).

Cohn, Henry; Umans, Christopher: **A Group-theoretic Approach to Fast Matrix Multiplication** 31-Oct-2003
(12 pages).

Pan, Victor Y. [City University of New York]: **Randomized Acceleration of Fundamental Matrix Computations** 23-Dec-2002
(13 pages).

Meppelink, David J.; Canning, James [University of Massachusetts Lowell]: **Matrix Multiplication Performance** 17-Dec-2002
(12 pages).

Raz, Ran [Weizmann Institute]: **On the Complexity of Matrix Product** 01-Dec-2002
(18 pages).

Pippenger, Nicholas: **Algebraic Complexity Theory** 17-Sep-2002
Algebraic complexity theory, the study of the minimum number of operations suficient to perform algebraic computations, is surveyed with emphasis on the general theory of bilinear forms and two of its applications: polynomial multiplication and matrix multiplication. Though by no means exhausting algebraic complexity theory, these topics illustrate well its development and its methods, and provide examples of its most striking successes. (8 pages).

Bläser, Markus [Lübeck University]: **On the complexity of the multiplication of matrices of small formats** 10-Jun-2002
We prove a lower bound of 2mn + 2n - m - 2 for the bilinear complexity of the multiplication of n x m-matrices with m x n-matrices using the substitution method (m >= n >

Alon, Noga; Galil, Zvi; Margalit, Oded; Naor, Moni [Tel Aviv University]: **Witnesses for Boolean Matrix Multiplication and for Shortest Paths** 20-Feb-2002
(10 pages).

Kaporin, Igor: **The Aggregation and Cancellation Techniques As a Practical Tool for Faster Matrix Multiplication** 01-Jan-2002
(38 pages).

Cormen, Thomas H.; Leiserson, Charles E.; Rivest, Ronald L.; Stein, Clifford: **Intrduction to Algorithms: Strassen's algorithm for matrix multiplication** 01-Sep-2001
Chapter 28.2, page 642, (5 pages).

Kolen, John F.; Bruce, Phillip Bruce [University of West Florida]: **Evolutionary Search for Matrix Multiplication Algorithms** 25-Jul-2001
This paper addresses the problem of algorithm discovery, via evolutionary search, in the context of matrix multiplication. The traditional multiplication algorithm requires O(n^3) multiplications for square matrices of order n. Strassen (Strassen 1969) discovered a recursive matrix multiplication algorithm requiring only seven multiplications at each level, resulting in a runtime of O(n^lgx7), or O(n2.81). We have been able to replicate this discovery using evolutionary search (Fogel 1995). The paper presents the representational schema, evaluation criteria, and evolution mechanisms employed during search. The most crucial decision was removing the determination of coefficients used to combine the product terms in the final addition steps from the search space and calculating them directly from the specified multiplications. Extending this methodology from 2 x 2 submatrices to algorithms using 3 × 3 decompositions is also discussed. (5 pages).

Aberdeen, Douglas; Baxter, Jonathan [Canberra University]: **Emmerald: a fast matrix-matrix multiply using Intel's SSE instructions** 08-Feb-2001
(17 pages).

Bügisser, Peter [Universität Zürich]: **Algebraische Komplexitätstheorie II: Schnelle Matrixmultiplikation und Kombinatorik** 07-Aug-2000
(16 pages).

Yagle, Andrew E. [University of Michigan]: **Fast Matrix Computations** 02-Jul-2000
(10 pages).

Kaporin, Igor [Russian Academy of Sciences, Moscow]: **A Practical Algorithm for Faster Matrix Multiplication** 25-Nov-1999
(14 pages).

Siek, Jeremy G. [University of Notre Dame]: **A Modern Framework for Portable High Performance Numerical Linear Algebra** 01-Apr-1999

This thesis describes a generic programming methodology for expressing data struc- tures, algorithms, and optimizations for numerical linear algebra. A high-performance implementation of this approach, the Matrix Template Library (MTL), is also described. The goal of the MTL is to facilitate development of higher-level libraries and applications for scientific computing. In addition, the programming techniques developed in this thesis are widely applicable and can be used to reduce development costs, improve readability, and improve the performance of many kinds of software. Portable high performance is a particular focus of the MTL. Flexible kernels were constructed that provide an automated tool for cross architecture performance portability. (297 pages).

Xiongda, Chen [Chinese Academy of Sciences]: **Implementation of Hopcroft-Kerr Algorithm on SR2201** 01-Jan-1999

(9 pages).

Thottethodi, Mithuna; Chatterjee, Siddhartha; Lebeck, Alvin R.: **Tuning Strassen's Matrix Multiplication for Memory Efficiency** 08-Jun-1998

trassen's algorithm for matrix multiplication gains its lower arithmetic complexity at the expense of reduced locality of reference, which makes it challenging to implement the algorithm efficiently on a modern machine with a hierarchical memory system. We report on an implementation of this algorithm that uses several unconventional techniques to make the algorithm memory-friendly. First, the algorithm internally uses a non-standard array layout known as Morton order that is based on a quad-tree decomposition of the matrix. Second, we dynamically select the recursion truncation point to minimize padding without affecting the performance of the algorithm, which we can do by virtue of the cache behavior of the Morton ordering. Each technique is critical for performance, and their combination as done in our code multiplies their effectiveness. Performance comparisons of our implementation with that of competing implementations show that our implementation often outperforms the alternative techniques (up to 25). However, we also observe wide variability across platforms and across matrix sizes, indicating that at this time, no single implementation is a clear choice for all platforms or matrix sizes. We also note that the time required to convert matrices to/from Morton order is a noticeable amount of execution time (5to 15). Eliminating this overhead further reduces our execution time.

Livingston: **Efficient Matrix Multiplication** 01-Jan-1998

Research proposal (3 pages).

Huang, Xiaohan; Pan, Victor Y. [City University of New York]: **Fast Rectangular Matrix Multiplication and Applications** 21-Jan-1997

(43 pages).

Huss-Lederman, Steven; Jacobson, Elaine M.; Johnson, J.R.; Tsao, Anna; Turnbull, Thomas: **Strassen's Algorithm for Matrix Multiplication:Modeling, Analysis, and Implementation** 15-Nov-1996

In this paper we report on the development of an efficient and portable implementation of Strassen's matrix multiplication algorithm for matrices of arbitrary size. Our implementation is designed to be used in place of DGEMM, the Level 3 BLAS matrix multiplication routine. Our code is designed so that efficient performance will be obtained for all matrix sizes and shapes and that the additional memory needed for temporary variables is minimized. Replacing DGEMM with our routine should provide significant performance gain for large matrices while providing the same performance for small matrices. We measure performance of our code on the IBM RS/6000, CRAY YMP C90, and CRAY T3D single processor, and offer comparisons to other codes. Our performance data reconfirms that Strassen's algorithm is practical for realistic size matrices. The usefulness of our implementation is demonstrated by replacing DGEMM with our routine in a large application code. (70 pages).

Huss-Lederman, Steven; Jacobson, Elaine M.; Johnson, Jeremy R.; Tsao, Anna; Turnbull, Thomas: **Implementation of Strassen's Algorithm for Matrix Multiplication** 01-Aug-1996

(27 pages).

Grayson, Brian; Shah, Ajay Pankaj; van, Robert A. [University of Texas at Austin]: **A High Performance Parallel Strassen Implementation** 13-Jun-1995

In this paper, we give what we believe to be the first high performance parallel implementation of Strassen's algorithm for matrix multiplication. We show how under restricted conditions, this algorithm can be implemented plug compatible with standard parallel matrix multiplication algorithms. Results obtained on a large Intel Paragon system show a 10-20reduction in execution time compared to what we believe to be the fastest standard parallel matrix multiplication implementation available at this time. (10 pages).

Knoght, Philip A. [University of Strathclyde]: **Fast Rectangular Matrix Multiplication and QR Decomposition** 01-May-1995

(13 pages).

Bini, Dario; Pan, Victor Y. [Birkhäuser]: **Polynomial and matrix computations. Fundamental algorithms. Vol.1** 01-Jan-1994

(416 pages).

Bini, Dario; Pan, Victor Y. [Birkhäuser]: **Polynomial and matrix computations. Fundamental algorithms. Vol.1** 01-Jan-1994

(416 pages).

Anderson, N. Anderson; Manley, D. Manley [Digital Equipment Coproration]: **A matrix extension of Winograd's inner product algorithm** 01-Feb-1993

(3 pages).

Strassen, Volker [Universität Konstanz]: **Algebra and Complexity** 01-Jul-1992

(16 pages).

Julian Laderman, Victo Y. Pan; Sha, Xuan-He: **On Practical Algorithms for Accelerated Matrix Multiplication** 01-Feb-1992
We present several bilinear algorithms for the acceleration of multiplication of n X n matrices that are superior to both the classical and Strassen's algorithm for moderate n (starting with n = 20). The Bini-Lotti result on the weak stability of bilinear algorithms applies to these algorithms. We present them in two equivalent versions, bilinear and trilinear. We also apply one of these algorithms over finite fields (or rings) of constants. Surprisingly, this enables us to decrease the bilinear complexity of n X n matrix multiplication (for 20 <= n <

Zhang, Xing [Western Carolina University]: **Choosing a Better Algorithm for Matrix Multiplication** 01-Jan-1991
This thesis conducts the study of three algorithms; the straightforward algorithm, Winograd's algorithm, Strassen's algorithm, their time complexities, and compares the three algorithms using graphs. The thesis also briefly describes two asymptotic improvements: Pan's of 1983 and Strassen's of 1986. (91 pages).

Huang, C.-H.; Johnson, J.R.; Johnson, R.W.: **A Tensor Product Formulation of Strassen's Matrix Multiplication Algorithm** 01-Feb-1990
(5 pages).

Bshouty, Nader H. [Technion - Israel Institute of Technology]: **A lower bound for matrix multiplication** 03-Nov-1988
(11 pages).

Bailey, David H. [NASA]: **Extra-High Speed Matrix Multiplication on the Cray-2** 01-May-1988
(7 pages).

Coppersmith, Don; Winograd, Shmuel [IBM]: **Matrix Multiplication via Arithmetic Progressions** 17-May-1987
(31 pages).

Makarov, O. M.: **Makarov-1987-A_non-commutative_algorithm_for_multiplying_5_x_5_matrices_using_one_hundred_multiplications** 01-Jan-1987
(3 pages). translated version

Johnson, Rodney W.; McLoughlin, Aileen M.: **Noncommutative Bilinear Algorithms for 3x3 Matrix Multiplication** 01-May-1986
New noncommutative bilinear algorithms for 3 x 3 matrix multiplication are presented. These have the same complexity, 23 essential multiplications, as the one discovered by Laderman, but are inequivalent to it. Equivalence here refers to a certain group of transformations all of which map noncommutative bilinear matrix-multiplication algorithms into other such algorithms; "inequivalent" means not related by a transformation in the group. This group has been studied by de Groote, who has shown for the case of 2 x 2 matrix multiplication with 7 essential multiplications that all such algorithms are equivalent to Strassen's. The new algorithms, by contrast, include infinitely many pairwise inequivalent algorithms. The computer search that led to the new algorithms is described. (9 pages).

Makarov, O. M.: **An algorithm for multiplying 3x3 matrices** 01-Jan-1986
(2 pages). in Russian language!

Pan, Victor Y.: **How can we speed up Matrix Multiplication?** 01-Jul-1984
(24 pages).

Coppersmith, Don: **Rapid Multiplication of Rectangular Matrices** 01-Aug-1982
(5 pages).

Alekseyev, Valery B. [Moscow State University]: **On the Complexity of Some Algorithms of Matrix Multiplication** 05-May-1982
(15 pages).

Schönhage, Arnold [Tübingen University]: **Partial and Total Matrix Multiplication** 01-Aug-1981
(22 pages).

Bini, Dario A. [Pisa University]: **Relations between exact and approximate bilinear Algorithms** 10-Apr-1979
(11 pages).

Brockettt, Roger W.; Dobkin, David [Harvard University]: **On the Optimal Evaluation of a Set of Bilinear Forms** 01-Jan-1978
(29 pages).

Sýkora, Ondrej [Slovak Academy of Sciences]: **A Fast Non-Commutative Algorithm for Matrix Multiplication** 01-Jan-1977
In the paper a non-commutative algorithm for the multiplication of two square matrices of order n is presented. The algorithm requires n3-(n-1)2 multiplications and n3-n2+ 11 (n-1)2 additions. The recursive application of the algorithm for matrices of order nk leads to O(klogn[n3-(n-1)2]n) operations to be executed.It is shown that some well-known algorithms are special cases of our algorithm. Finally, an improvement of the algorithm is given for matrices of order 5. (6 pages).

Laderman, Julian D.: **A Noncommutative Algorithm for Multiplying 3 x 3 Matrices Using 23 Multiplications** 09-Oct-1975
(3 pages).

Makarov, O. M.: **Makarov - 1975 - The connection between the fast transform algorithms of Fourier and Hadamard and the algorithms of Karacuba, Strassen and Winograd** 01-Jan-1975
(11 pages). in Russian language! English version: http://www.sciencedirect.com/science/article/pii/0041555375900993

Hopcroft, John E.; Musinski, J. [Cornell University]: **Duality Applied to the Complexity of Matrix Multiplications and other Bilinear Forms** 01-Jan-1973

The paper considers the complexity of bilinear forms in a noncommutative ring. The dual of a computation is defined and applied to matrix multiplication and other bilinear forms. It is shown that the dual of an optimal computation gives an optimal computation for a dual problem. An nxm by mxp matrix product is shown to be the dual of an nxp by pxm or an mxn by nxp matrix product implying that each of the matrix products requires the same number of multiplications to compute. Finally an algorithm for computing a single bilinear form over a noncommutative ring with a minimum number of multiplications is derived by considering a dual problem. (15 pages).

Probert, Robert L.: **On the Complexity of Symmetric Computations** 01-Jan-1973

(31 pages).

Brent, R. P. [Stanford University]: **Algorithms for Matrix Multiplication** 01-Mar-1970

Strassen's and Winograd's algorithms for n × n matrix multiplication are investigated and compared with the normal algorithm. The normal algorithm requires n3 + O(n2) multiplications and about the same number of additions. Winograd's algorithm almost halves the number of multiplications at the expense of more additions. Strassen's algorithm reduces the total number of operations to O(n2.82) by recursively multiplying 2n × 2n matrices using seven n × n matrix multiplications. Floating-point error bounds are obtained for both Strassen's and Winograd's methods. It is shown that Strassen's method satisfies a certain numerical stability property (albeit weaker than that satisfied by the normal method); and that scaling is essential for numerical accuracy using Winograd's method. In practical cases, for moderate n, Winograd's method appears to be slightly faster than the other two methods, but the gain is, at most, about 20 percent. Strassen's method should be faster for sufficiently large n, and this will be important in the future as memory sizes increase. An attempt to generalize Strassen's method is described. (55 pages).

Hopcroft, John .E.; Kerr, Leslie Robert [Cornell University]: **On Minimizing the Number of Multiplications Necessary for Matrix Multiplication** 01-Sep-1969

This paper develops an algorithm to multiply a px2 matrix by a 2xn matrix in $\lceil (3pn+max(n,p))/2 \rceil$ multiplications for matrix multiplication without commutativity. The algorithm minimizes the number of multiplications for matrix multiplication without commutativity for the special cases p=1 or 2, n=1,2, $\cdots$ and p = 3, n = 3. It is shown that with commutativity fewer multiplications are required. (34 pages).

Waksman, Abraham [Stanford Research Institute]: **On Winograd's Algorithm for Inner Products** 04-Aug-1969

(2 pages).

Strassen, Volker [Zurich University]: **Gaussian Elimination is not optimal** 12-Dec-1968

(3 pages).

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