Articles and book chapters on matrix multiplication.

Some documents and comments should only be used internally. Confidential documents are marked as such. Please take note of legal notices. In case something is missing, please notify Axel Kemper

Note 1: There is a "find" button at the end of the page. It allows you to list all the references which have a keyword in common.

Note 2: Click the small flag at the bottom to see this page in German.

Matrix Mutliplication and SAT References
Articles and book chapters on matrix multiplication and Boolean satisfiability. Some documents and comments should only be used internally. Confidential documents are marked as such. Please take note of legal notices. In case something is missing, please notify Axel Kemper

Note 1: There is a "find" button at the end of the page. It allows you to list all the references which have a keyword in common.

Note 2: Click the small flag at the bottom to see this page in German.

Grochow, Joshua A.; Moore, Christopher: **Designing Strassen's Algorithm** — 30-Aug-2017
In 1969, Strassen shocked the world by showing that two n × n matrices could be multiplied in time asymptotically less than O(n 3). While the recursive construction in his algorithm is very clear, the key gain was made by showing that 2 × 2 matrix multiplication could be performed with only 7 multiplications instead of 8. The latter construction was arrived at by a process of elimination and appears to come out of thin air. Here, we give the simplest and most transparent proof of Strassen's algorithm that we are aware of, using only a simple unitary 2-design and a few easy lines of calculation. Moreover, using basic facts from the representation theory of finite groups, we use 2-designs coming from group orbits to generalize our construction to all n = 2 (although the resulting algorithms aren't optimal for n = 3). (10 pages).

Sedoglavic, Alexandre: **A non-commutative algorithm for multiplying 5 × 5 matrices using 99 multiplications** — 24-Jul-2017
(7 pages).

Karstadt, Elaye; Schwartz, Oded [The Hebrew University of Jerusalem]: **Matrix Multiplication, a Little Faster** — 24-Jul-2017
Strassen's algorithm (1969) was the first sub-cubic matrix multiplication algorithm. Winograd (1971) improved its complexity by a constant factor. Many asymptotic improvements followed. Unfortunately, most of them have done so at the cost of very large, often gigantic, hidden constants. Consequently, Strassen-Winograd's O(n^lb(7)) algorithm often outperforms other matrix multiplication algorithms for all feasible matrix dimensions. The leading coefficient of Strassen-Winograd's algorithm was believed to be optimal for matrix multiplication algorithms with 2 × 2 base case, due to a lower bound of Probert (1976). Surprisingly, we obtain a faster matrix multiplication algorithm, with the same base case size and asymptotic complexity as Strassen-Winograd's algorithm, but with the coefficient reduced from 6 to 5. To this end, we extend Bodrato's (2010) method for matrix squaring, and transform matrices to an alternative basis. We prove a generalization of Probert's lower bound that holds under change of basis, showing that for matrix multiplication algorithms with a 2 × 2 base case, the leading coecient of our algorithm cannot be further reduced, hence optimal. We apply our technique to other Strassen-like algorithms, improving their arithmetic and communication costs by signifficant constant factors. (10 pages).

Chiantini, Luca; Hauenstein, Jonathan D.; Ikenmeyer, Christian; Landsberg, J.M.; Ottaviani, Giorgio: **Polynomials and the exponent of matrix multiplication** — 15-Jun-2017
We define tensors, corresponding to cubic polynomials, which have the same exponent omega as the matrix multiplication tensor. In particular, we study the symmetrized matrix multiplication tensor sM

Sedoglavic, Alexandre: **Laderman matrix multiplication algorithm can be constructed using Strassen algorithm and related tensor's isotropies** — 11-May-2017
(14 pages).

Smirnov, Alexey: **Several bilinear algorithms for matrix multiplication** — 01-Jan-2017
(21 pages).

Dumas, Jean-Guillaume; Pan, Victor Y.: **Fast Matrix Multiplication and Symbolic Computation** — 17-Dec-2016
The complexity of matrix multiplication (hereafter MM) has been intensively studied since 1969, when Strassen surprisingly decreased the exponent 3 in the cubic cost of the straightforward classical MM to log_2(7) ~ 2.8074. Applications to some fundamental problems of Linear Algebra and Computer Science have been immediately recognized, but the researchers in Computer Algebra keep discovering more and more applications even today, with no sign of slowdown. We survey the un?nished history of decreasing the exponent towards its information lower bound 2, recall some important techniques discovered in this process and linked to other fields of computing, reveal sample surprising applications to fast computation of the inner products of two vectors and summation of integers, and

discuss the curse of recursion, which separates the progress in fast MM into its most acclaimed and purely theoretical part and into valuable acceleration of MM of feasible sizes. Then, in the second part of our paper, we cover fast MM in realistic symbolic computations and discuss applications and implementation of fast exact matrix multiplication. We first review how most of exact linear algebra can be reduced to matrix multiplication over small finite fields. Then we highlight the differences in the design of approximate and exact implementations of fast MM, taking into account nowadays processor and memory hierarchies. In the concluding section we comment on current perspectives of the study of fast MM. (32 pages).

Grochow, Joshua A.; Moore, Cristopher: **Matrix multiplication algorithms from group orbits** 12-Dec-2016
We show how to construct highly symmetric algorithms for matrix multiplication. In particular, we consider algorithms which decompose the matrix multiplication tensor into a sum of rank-1 tensors, where the decomposition itself consists of orbits under some ?nite group action. We show how to use the representation theory of the corresponding group to derive simple constraints on the decomposition, which we solve by hand for n = 2 , 3 , 4 , 5, recovering Strassen?s algorithm (in a particularly symmetric form) and new algorithms for larger n . While these new algorithms do not improve the known upper bounds on tensor rank or the matrix multiplication exponent, they are beautiful in their own right, and we point out modifications of this idea that could plausibly lead to further improvements. Our constructions also suggest further patterns that could be mined for new algorithms, including a tantalizing connection with lattices. In particular, using lattices we give the most transparent proof to date of Strassen?s algorithm; the same proof works for all n , to yield a decomposition with n^3 - n + 1 terms. (16 pages).

Huang, Jianyu; Rice, Leslie; Matthews, Devin A.; van, Robert A. [The University of Texas at Austin,]: **Generating Families of Practical Fast Matrix Multiplication Algorithms** 03-Nov-2016
Matrix multiplication (GEMM) is a core operation to numerous scientific applications. Traditional implementations of Strassen-like fast matrix multiplication (FMM) algorithms often do not perform well except for very large matrix sizes, due to the increased cost of memory movement, which is particularly noticeable for non-square matrices. Such implementations also require considerable workspace and modifications to the standard BLAS interface. We propose a code generator framework to automatically implement a large family of FMM algorithms suitable for multiplications of arbitrary matrix sizes and shapes. By representing FMM with a triple of matrices J U, V, W K that capture the linear combinations of submatrices that are formed, we can use the Kronecker product to define a multi-level representation of Strassen-like algorithms. Incorporating the matrix additions that must be performed for Strassen-like algorithms into the inherent packing and micro-kernel operations inside GEMM avoids extra workspace and reduces the cost of memory movement. Adopting the same loop structures as high-performance GEMM implementations allows parallelization of all FMM algorithms with simple but efficient data parallelism without the overhead of task parallelism. We present a simple performance model for general FMM algorithms and compare actual performance of 20+ FMM algorithms to modeled predictions. Our implementations demonstrate a performance benefit over conventional GEMM on single core and multicore systems. This study shows that Strassen-like fast matrix multiplication can be incorporated into libraries for practical use. (17 pages).

Chiantini, Luca; Ikenmeyer, Christian; Landsberg, J.M.; Ottaviani, Giorgio: **The Geometry of Rank Decompositions of Matrix Multiplication I - 2x2 Matrices** 25-Oct-2016
This is the first in a series of papers on rank decompositions of the matrix multiplication tensor. In this paper we: establish general facts about rank decompositions of tensors, describe potential ways to search for new matrix multiplication decompositions, give a geometric proof of the theorem of [3] establishing the symmetry group of Strassen's algorithm, and present two particularly nice subfamilies in the Strassen family of decompositions. (9 pages).

Tichavský, Petr; Phan, Anh-Huy; Cichocki, Andrzej: **Numerical CP decomposition of some difficult tensors** 24-Aug-2016
In this paper, a numerical method is proposed for canonical polyadic (CP) decomposition of small size tensors. The focus is primarily on decomposition of tensors that correspond to small matrix multiplications. Here, rank of the tensors is equal to the smallest number of scalar multiplications that are necessary to accomplish the matrix multiplication. The proposed method is based on a constrained Levenberg-Marquardt optimization. Numerical results indicate the rank and border ranks of tensors that correspond to multiplication of matrices of the size 2×3 and 3×2, 3×3 and 3×2, 3×3 and 3×3, and 3×4 and 4×3. The ranks are 11, 15, 23 and 29, respectively. In particular, a novel algorithm for computing product of matrices of the sizes 3×4 and 4×3 using 29 multiplications is presented. (9 pages).

Tichavský, Petr; Phan, Anh-Huy; Cichocki, Andrzej: **Numerical CP decomposition of some difficult tensors** 24-Aug-2016
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Tichavský, Petr; Phan, Anh Huy; Cichocki, Andrzej: **Numerical CP Decomposition of Some Difficult Tensors** 04-May-2016
(11 pages).

Elser, Veit [Cornell University]: **A network that learns Strassen multiplication** 26-Jan-2016

We study neural networks whose only non-linear components are multipliers, to test a new training rule in a context where the precise representation of data is paramount. These networks are challenged to discover the rules of matrix multiplication, given many examples. By limiting the number of multipliers, the network is forced to discover the Strassen multiplication rules. This is the mathematical equivalent of finding low rank decompositions of the n × n matrix multiplication tensor, Mn . We train these networks with the conservative learning rule, which makes minimal changes to the weights so as to give the correct output for each input at the time the input-output pair is received. Conservative learning needs a few thousand examples to find the rank 7 decomposition of M2 , and 10^5 for the rank 23 decomposition of M3 (the lowest known). High precision is critical, especially for M3 , to discriminate between true decompositions and "border aproximations". (15 pages).

Pan, Victor [City University of New York (CUNY)]: **Matrix Multiplication, Trilinear Decompositions, APA Algorithms, and Summation** 10-Dec-2015
(18 pages).

Mourouzis, Theodosis: **Multiplicative Complexity Reductions in Cryptography and Cryptanalysis** 02-Dec-2015
(77 slides).

Burichenko, Vladimir P. [Institute of mathematics of National Academy of Sciences of Belarus]: **Symmetries of matrix multiplication algorithms. I** 05-Aug-2015
(53 pages).

Minz, Jacob: **Derivation of Strassen's Algorithm for the multiplication of 2 x 2 matrices** 04-May-2015
Strassen's multiplication algorithm for the multiplication of matrices using seven multiplication instead of the naive 8 multiplication heralded a new era of research into asymptotically more efficient algorithms. It showed that the multiplication of two n x n matrices could be achieved in less than $O(n^3)$ time. In this blog, we attempt to present a derivation of this seminal work, which only requires a modest background in linear algebra. (3 pages).

Volte, Emmanuel: **Miroirs, Cubes et Feistel Dissymétriques** 18-Apr-2015
(143 pages).

Bläser, Markus: **ExplicitTensors** 26-Mar-2015
This is an expository article the aim of which is to introduce interested students and researchers to the topic of tensor rank, in particular to the construction of explicit tensors of high rank. We try to keep the mathematical concepts and language used as simple as possible to address a broad audience. This article is thought to be an appetizer and does not provide by any means a complete coverage of this topic. (13 pages).

Kemper, Axel: **Verfahren von Makarov zur Multiplikation von 5x5 Matrizen** 01-Jan-2015
(19 pages).

Pan, Victor Y. [City University of New York]: **Matrix Multiplication, Trilinear Decompositions, APA Algorithms, and Summation** 18-Dec-2014
Matrix multiplication (hereafter we use the acronym MM) is among the most fundamental operations of modern computations. The efficiency of its performance depends on various factors, in particular vectorization, data movement and arithmetic complexity of the computations, but here we focus just on the study of the arithmetic cost and the impact of this study on other areas of modern computing. In the early 1970s it was expected that the straightforward cubic time algorithm for MM will soon be accelerated to enable MM in nearly quadratic arithmetic time, with some far fetched implications. While pursuing this goal the mainstream research had its focus on the decrease of the classical exponent 3 of the complexity of MM towards its lower bound 2, disregarding the growth of the input size required to support this decrease. Eventually, surprising combinations of novel ideas and sophisticated techniques enabled the decrease of the exponent to its benchmark value of about 2.38, but the supporting MM algorithms improved the straightforward one only for the inputs of immense sizes. Meanwhile, the communication complexity, rather than the arithmetic complexity, has become the bottleneck of computations in linear algebra. This development may seem to undermine the value of the past and future research aimed at the decrease of the arithmetic cost of MM, but we feel that the study should be reassessed rather than closed and forgotten. We review the old and new work in this area in the present day context, recall some major techniques introduced in the study of MM, discuss their impact on the modern theory and practice of computations for MM and beyond MM, and link one of these techniques to some simple algorithms for inner product and summation. (17 pages).

Ambainis, A.; Filmus, Y. [Electronic Colloquium on Computational Complexity, Report No. 154 (2014)]: **Ambainis - 2014 - Fast Matrix Multiplication - Limitations of the Laser Method** 20-Nov-2014
(39 pages).

Ballard, Grey [Sandia National Laboratories]: **Communication-Avoiding Algorithms and Fast Matrix Multiplication** 29-Sep-2014
(78 slides).

Benson, Austin; Ballard, Grey: **A Framework for Practical fast Matrix Multiplication** 26-Sep-2014
(32 slides).

Benson, Austin R.; Ballard, Grey: **A Framework for Practical Parallel Fast Matrix Multiplication** 09-Sep-2014
(14 pages).

Burichenko, Vladimir P. [Institute of mathematics of Academy of Sciences of Belarus]: **On symmetries of the** 26-Aug-2014

Strassen algorithm
(16 pages).

Bläser, Markus [Saarland University]: **Fast matrix multiplication and related problems** 09-Jul-2014
(36 slides).

Williams, Virginia Vassilevska [Stanford University]: **Multiplying matrices in O(n^2.373) time** 01-Jul-2014
(73 pages).

Ottaviani, Giorgio [Universit`a di Firenze]: **A brief survey on tensor rank and tensor decomposition, from a geometric perspective. Part I** 02-Jun-2014
(76 slides).

Boyer, Brice; Dumas, Jean-Guillaume: **Boyer-Dumas - 2014 - Matrix multiplication over word-size prime Fields using Bini?s approximate formula** 06-May-2014
(14 pages).

Ottaviani, Giorgio [Università di Firenze]: **Complexity of Matrix Multiplication and Tensor Rank** 27-Feb-2014
KIAS, Seoul The naive way to multiply two matrixes is not computationally optimal for matrices of large size. The complexity of the algorithm of matrix multiplication coincides asymptotically with the rank of the matrix multiplication tensor. We present some basic results in this area and a lower bound on this rank, obtained in collaboration with J.M. Landsberg, by using techniques from algebraic geometry and representation theory. (37 slides).

Kemper, Axel: **Verfahren von Makarov zur Multiplikation von 5x5 Matrizen** 06-Jan-2014
This note (in German) describes Makarov's method to multiply 5x5 matrices using 100 elemtary multiplications. While Makarov only showed the existance of such an algorithm, the note describes the algorithm explicitely using a Yacas CAS script. (17 pages).

Sauter, S. [Universität Zürich]: **Numerical Tensor Calculus** 19-Dec-2013
(110 pages).

Daleo, Noah S.; Hauenstein, Jonathan D. [North Carolina State University]: **Daleo - Hauenstein - 2013 - A numerical approach to tensor decomposition** 25-Nov-2013
(38 slides).

Laitinen, Tero; Junttila, Tommi; Niemelä, Ilkka [Aalto University]: **Simulating Parity Reasoning (extended version)** 18-Nov-2013
Propositional satisfiability (SAT) solvers, which typically operate using conjunctive normal form (CNF), have been successfully applied in many domains. However, in some application areas such as circuit verification, bounded model checking, and logical cryptanalysis, instances can have many parity (xor) constraints which may not be handled efficiently if translated to CNF. Thus, extensions to the CNF-driven search with various parity reasoning engines ranging from equivalence reasoning to incremental Gaussian elimination have been proposed. This paper studies how stronger parity reasoning techniques in the DPLL(XOR) framework can be simulated by simpler systems: resolution, unit propagation, and parity explanations. Such simulations are interesting, for exam-ple, for developing the next generation SAT solvers capable of handling parity constraints efficiently. (37 pages).

Landsberg, J.M.: **New Lower Bounds for the Rank of Matrix Multiplication** 29-Oct-2013
The rank of the matrix multiplication operator for nxn matrices is one of the most studied quantities in algebraic complexity theory. I prove that the rank is at least n^2-o(n^2). More precisely, for any integer p\leq n -1, the rank is at least (3- 1/(p+1))n^2-(1+2p\binom{2p} {p-1})n. The previous lower bound, due to Blaser, was 5n^2/2-3n (the case p=1). The new bounds improve Blaser's bound for all n>84. I also prove lower bounds for rectangular matrices significantly better than the the previous bound. (6 pages).

Landsberg, J.M.; Ottaviani, Giorgio: **New Lower Bounds for the Border Rank of Matrix Multiplication** 02-Jun-2013
The border rank of the matrix multiplication operator for n × n matrices is a standard measure of its complexity. Using techniques from algebraic geometry and representation theory, we show the border rank is at least $2n^2$ - n. Our bounds are better than the previous lower bound (due to Lickteig in 1985) of $3/2n^2$ + n/2 - 1 for all n >= 3. The bounds are obtained by finding new equations that bilinear maps of small border rank must satisfy, i.e., new equations for secant varieties of triple Segre products, that matrix multiplication fails to satisfy. (11 pages).

Vannieuwenhoven, Nick [KU Leuven]: **On generalizing the Schmidt-Eckart-Young theorem to tensors** 20-May-2013
(53 pages).

Oh, Jinsoo; Kim, Jin; Moon, Byung-Ro: **On the inequivalence of bilinear algorithms for 3 × 3 matrix multiplication** 14-May-2013
Since Laderman showed an algorithm for 3 × 3 matrix multiplication using 23 scalar multiplications, Johnson and McLoughlin used a numerical optimization and human controlled method to give two parameterized algorithms in which the coefficients are rational numbers. The algorithms are inequivalent to Laderman?s one with respect to the transformation introduced by de Groote. We present a simple and fast numerical heuristic for finding valid algorithms. Then we show that many of the obtained algorithms are inequivalent to the published ones. (6 pages).

Hart, Sarah; Hedtke, Ivo; Müller-Hannemann, Matthias; Murthy, Sandeep: **A Fast Search Algorithm for [m, m, m] Triple Product Property Triples Aand An Application for 5 × 5 Matrix Multiplication** 01-May-2013

We present a new fast search algorithm for

Smirnov, A. V.: **The Bilinear Complexity and Practical Algorithms for Matrix Multiplication** 19-Mar-2013
A method for deriving bilinear algorithms for matrix multiplication is proposed. New estimates for the bilinear complexity of a number of problems of the exact and approximate multiplication of rectangular matrices are obtained. In particular, the estimate for the boundary rank of multiplying 3 × 3 matrices is improved and a practical algorithm for the exact multiplication of square n × n matrices is proposed. The asymptotic arithmetic complexity of this algorithm is O(n^2.7743). (15 pages). Russion version: http://www.mathnet.ru/php/archive.phtml?wshow=paper&jrnid=zvmmf&&option_lang=eng

Hedtke, Ivo [Martin-Luther-University Halle-Wittenberg]: **Hedtke - 2013 - (Group-theoretic) Fast Matrix Multiplication** 11-Mar-2013
(25 slides).

Bläser, Markus [Universität des Saarlandes]: **Fast matrix multiplication** 06-Mar-2013
We give an overview of the history of fast algorithms for matrix multiplication. Along the way, we look at some other fundamental problems in algebraic complexity like polynomial evaluation. This exposition is self-contained. To make it accessible to a broad audience, we only assume a minimal mathematical background: basic linear algebra, familiarity with polynomials in several variables over rings, and rudimentary knowledge in combinatorics should be sufficient to read (and understand) this article. This means that we have to treat tensors in a very concrete way (which might annoy people coming from mathematics), occasionally prove basic results from combinatorics, and solve recursive inequalities explicitly (because we want to annoy people with a background in theoretical computer science, too). (59 pages).

Murat Cenk, M. Anwar Hasan [University of Waterloo]: **On the Arithmetic Complexity of Strassen-Like Matrix Multiplications** 24-Feb-2013
(20 pages).

Courtois, Nicolas T. [University College London]: **Multiplicative Complexity** 28-Jan-2013
(105 slides).

Filmus, Yuval: **Matrix Multiplication 2** 14-Jan-2013
(14 pages).

Bläser, Markus [Saarland University]: **Fast matrix multiplication and related problems - Slides** 01-Jan-2013
(36 slides).

Massri, César [University of Buenos Aires]: **Solving a sparse systems using linear algebra** 15-Nov-2012
We generalize a method to compute the solutions of a system of polynomial equations with finitely many solutions. We solve this problem without the need of Gröbner bases. (15 pages).

Petersen, Kaare Brandt; Pedersen, Michael Syskind: **The Matrix Cookbook** 15-Nov-2012
(72 pages).

Ottaviani, Giorgio [Università di Firenze]: **Tensor decomposition and tensor rank from the point of view of Classical Algebraic Geometry** 26-Sep-2012
(57 pages).

Cohn, Henry; Umans, Christopher: **Fast matrix multiplication using coherent configurations** 27-Jul-2012
(18 pages).

Joó, A.; Ekárt, A.; Neirotti, J. P. [Aston University, Birmingham]: **Genetic algorithms for discovery of matrix multiplication methods** 07-Jul-2012
We present a parallel genetic algorithm for nding matrix multiplication algorithms. For 3 × 3 matrices our genetic algorithm successfully discovered algorithms requiring 23 multiplications, which are equivalent to the currently best known human-developed algorithms. We also studied the cases with less multiplications and evaluated the suitability of the methods discovered. Although our evolutionary method did not reach the theoretical lower bound it led to an approximate solution for 22 multiplications. (7 pages).

Gall, Francois Le: **Faster Algorithms for Rectangular Matrix Multiplication** 09-Apr-2012
(37 pages).

N.N.: **Note on the Strassen formulas** 07-Apr-2012
The Strassen algorithm for matrix multiplication is well explained in the textbook, see CLRS 4.2. However, all these formulas for P1, . . . , P7 look puzzling: it's a complete mystery where they came from. The purpose of this note is to make the algebra slightly less mysterious by revealing some symmetry. (2 pages).

T.Courtois, Nicolas; Hulme, Daniel; Mourouzis, Theodosis [University College London]: **Multiplicative Complexity And Solving Generalized Brent Equations With SAT Solvers** 31-Mar-2012
(6 pages).

Courtois, Nicolas T.; Hulme, Daniel; Mourouzis, Theodosis [University College London; NP-Complete Ltd.]: **Multiplicative Complexity Gate Complexity Cryptography and Cryptanalysis** 18-Mar-2012
(75 slides).

Filmus, Yuval: **Matrix Multiplication 1** 02-Feb-2012
(13 pages).

Lipton, R.J.: **A Brief History Of Matrix Product** 01-Feb-2012
(5 pages).

Holtz, Olga [University of California at Berkeley]: **Fast and stable matrix multiplication** 31-Dec-2011
(44 slides).

Williams, Virginia Vassilevska [University of California, Berkeley]: **Breaking the Coppersmith-Winograd barrier** 30-Nov-2011
We develop new tools for analyzing matrix multiplication constructions similar to the Coppersmith-Winograd construction, and obtain a new improved bound on omega < 2.3727. (72 pages).

Courtois, Nicolas T.; Bard, Gregory V.; Hulme, Daniel: **A New General-Purpose Method to Multiply 3x3 Matrices Using Only 23 Multiplications** 19-Aug-2011
(10 pages).

Joó, András; Ekárt, Anikó; Neirotto, Juan [Aston University Mirmingham]: **Genetic Algorithms for Fast Matrix Multiplication** 14-Jul-2011
(9 slides). https://research.aston.ac.uk/portal/files/244971/letter-18-04.pdf

Hedtke, Ivo: **Methods of Matrix Multiplication: An Overviw of several methods and their implementation** 08-Jun-2011
In this overview article we present several methods for multiplying matrices and the implementation of these methods in C. Also a little test program is given to compare their running time and the numerical stability. The methods are: naive method, naive method working on arrays, naive method with the KAHAN trick, three methods with loop unrolling, winograd method and the scaled variant, original STRASSEN method and the STRASSEN-WINOGRAD variant. (25 pages).

Landsberg, J.M.: **Tensors: Geometry and Applications** 14-May-2011
Tensors are ubiquitous in the sciences. One reason for their ubiquity is that they provide a useful way to organize data. Geometry is a powerful tool for extracting information from data sets, and a beautiful subject in its own right. This book has three intended uses: as a classroom textbook, a reference work for researchers, and a research manuscript. (83 pages).

Hedtke, Ivo [Jena University]: **Darstellungstheoretische Ansätze in der schnellen Matrixmultiplikation** 13-May-2011
Diploma thesis (58 pages). Slides: http://www.ihmm.de/documents/grueppchen2012.pdf

Hedtke, Ivo: **A Note on the Group-Theoretic Approach to Fast Matrix Multiplication** 11-May-2011
(5 pages).

Hedtke, Ivo; Murthy, Sandeep: **Search and Test Algorithms for Triple Product Property Triples** 11-May-2011
In 2003 COHN and UMANS introduced a group-theoretic approach to fast matrix multiplication. This involves finding large subsets of a group G satisfying the Triple Product Property (TPP) as a means to bound the exponent ? of matrix multiplication. We present two new characterizations of the TPP, which are useful for theoretical considerations and for TPP test algorithms. With this we describe all known TPP tests and implement them in GAP algorithms. We also compare their runtime. Furthermore we show that the search for subgroup TPP triples of nontrivial size in a nonabelian group can be restricted to the set of all nonnormal subgroups of that group. Finally we describe brute-force search algorithms for maximal subgroup and subset TPP triples. In addition we present the results of the subset brute-force search for all groups of order less than 25 and selected results of the subgroup brute-force search for 2-groups, SL(n,q) and PSL(2,q). (14 pages).

Hedtke, Ivo: **Strassen's Matrix Multiplication Algorithm for Matrices of Arbitrary Order** 11-May-2011
(8 pages).

Drevet, Charles-Éric; Islam, Md. Nazrul; Schost, Éric: **Optimization Techniques for Small Matrix Multiplication** 06-Oct-2010
(1 Poster).

Bodrato, Marco: **Bodrato - 2010 - Strassen-like Matrix Multiplication for Squares** 30-Jul-2010
(8 pages).

Bodrato, Marco: **Strassen-like-Matrix Multiplication suited for Squaring and Higher Power Computation** 28-Jul-2010
(28 slides).

Drevet, Charles-Éric; Islam, Md. Nazrul; Schost, Eric [Universiyt of Western Ontario]: **Optimization Techniques for Small Matrix Multiplication** 14-May-2010
(32 pages).

Smith, Warren D.: **Fast matrix algorithms and multiplication formulae** 18-Apr-2010
Research proposal: Contains a table of 25 approximate matrix multiplication algorithms, many of them better than the Strassen algorithms (example: 5x5x5 with 90 products) (17 pages). Note: Warren D. Smith could not be found in internet searches after this proposal.

Kundeti, Vamsi [University of Connecticut]: **A Simplified Proof For The Application Of Freivalds' Technique to Verify Matrix Multiplication** 26-Mar-2010
(2 pages).

Pospelov, Alexey [Saarland University]: **Bounds for Bilinear Complexity of Noncommutative Group Algebras** 24-Mar-2010
We study the complexity of multiplication in noncommutative group algebras which is closely related to the complexity of matrix multiplication. We characterize such semisimple group algebras of the minimal bilinear complexity and show nontrivial lower bounds for the rest of the group algebras. These lower bounds are built on the top of Bläser's results for semisimple algebras and algebras with large radical and the lower bound for arbitrary associative algebras due to Alder and Strassen. We also show subquadratic upper bounds for all group algebras turning into "almost linear" provided the exponent of matrix multiplication equals 2. (20 pages).

va de de Geijn, Robert A.; Watts, Jerrell [University of Texas at Austin, Caltech]: **de Geijn-Watts - 2010 - SUMMA Scalable Universal Matrix Multiplication Algorithm** 02-Mar-2010
(19 pages).

Islam, Md. Nazrul [University of Western Ontario, Canada]: **A Generalization of Waksman's Matrix Multiplication** 12-Feb-2010
(3 pages).

Islam, Md. Nazrul: **Formulas for Hopcroft-Kerr's Matrix Multiplication Algorithm** 12-Feb-2010
(3 pages).

Srimani, P.K.; Rani, G. Vakula: **Srimani-Rani -2010 - A Comparative Study of Fast Matrix Multiplication Methods (FMMM) by using Vedic Algorithms** 10-Feb-2010
(6 pages).

Anderson, Matthew; Barman, Siddharth: **The Coppersmith-Winograd Matrix Multiplication Algorithm** 06-Dec-2009
(11 pages).

Oh, Seunghyun; Moon, Byung-Ro: **Strassen's Algorithm Revisited by Genetic Search** 02-Dec-2009
(6 pages).

Loos, Sarah M.; Wise, David S.: **Strassen's Matrix Multiplication Relabeled** 01-Dec-2009
A very simple recasting of this classic 7-multiplication recursion improves its time performance for rectangular matrices of order n when n is not a power of two. No time is lost in the rare cases when n = 2^p. A similar relabeling applies, as well, to Winograd's 15-addition variant, for which experiments on n × 2n × n products show an average 25 3472368me improvement. (7 pages).

Williams, Virginia Vassilevska; Williams, Ryan [Princeton University]: **Triangle Detection Versus Matrix Multiplication** 05-Nov-2009
(18 pages).

Bläser, Markus [Saarland University]: **Complexity of Bilinear Problems - Computational Complexity** 07-Oct-2009
(46 pages).

Holtz, Olga; Shomron, Noam: **Computational Complexity and Numerical Stability of Linear Problems** 12-Sep-2009
We survey classical and recent developments in numerical linear algebra, focusing on two issues: computational complexity, or arithmetic costs, and numerical stability, or performance under roundoff error. We present a brief account of the algebraic complexity theory as well as the general error analysis for matrix multiplication and related problems. We emphasize the central role played by the matrix multiplication problem and discuss historical and modern approaches to its solution. (16 pages).

Islam, Md. Nazrul [University of Western Ontario]: **Optimization Techniques For Matrix Multiplication** 27-Aug-2009
(95 pages).

Eve, James [Newcastle University]: **On O(n^2 log n) algorithms for n×n matrix operations** 01-Aug-2009
(42 pages).

Färnqvist, Tommy [Linköping University]: **Lower Bounds for Matrix Multiplication** 09-Jun-2009
Master's Thesis Proposal (2 pages).

Orem, Hendrik [Harvey Mudd College]: **Fast Matrix Multiplication via Group Actions** 01-May-2009
Recent work has shown that fast matrix multiplication algorithms can be constructed by embedding the two input matrices into a group algebra, ap-plying a generalized discrete Fourier transform, and performing the multiplication in the Fourier basis. Developing an embedding that yields a matrix multiplication algorithm with running time faster than naive matrix multiplication leads to interesting combinatorial problems in group theory. The crux of such an embedding, after a group G has been chosen, lies in finding a triple of subsets of G that satisfy a certain algebraic relation. I show how the process of finding such subsets can in some cases be greatly simplified by considering the action of the group G on an appropriate set X. In particular, I focus on groups acting on regularly branching trees. (41 pages).

Bowen, Richard Strong; Chen, Bob; Orem, Hendrik; Schaardenburg, Martijn van [Harvey Mudd College]: **Group-Theoretic Partial Matrix Multiplication** 13-Feb-2009
(14 pages).

Mayr, Ernst W. [Technical University Munich]: **Matrixmultiplikation à la Strassen** 12-Jan-2009
(9 slides).

Bard, Gregory V. [Fordham University]: **New Practical Strassen-like Approximate Matrix-Multiplication Algorithms found via solving a System of Cubic Equations** 15-Dec-2008
(17 pages).

Petersen, Kaare Brandt; Pedersen, Michael Syskind: **The Matrix Cookbook** 14-Nov-2008
(71 pages).

Bard, Gregory V.: **A Practical Algorithm for Massively-Parallel Dense Matrix Multiplication in time n 2.777** 29-Oct-2008
(72 slides).

Mézard, Marc; Montanari, Andrea: **Linear Equations with Boolean Variables** 22-Sep-2008
Solving a system of linear equations over a finite field is arguably one of the most fundamental operations in mathematics. This chapter considers a specific ensemble of random linear systems over Boolean variables, named XORSAT, and discusses the structure of its set of solutions. In large instances, the affine subspace of solutions can exhibit a remarkably rich geometrical structure. When the ratio of equations to variables is increased, the system first gets into an intermediate phase where solutions cluster in many well separated regions of the hypercube. Then it encounters a second phase transition and gets into an 'UNSAT' phase where the probability of existence of a solution vanishes. The study uses belief propagation equations, and a combinatorial analysis of the 2-core in the associated factor graph. (141 pages).

D'Alberto, Paolo; Nicolau, Alexandru: **Adaptive Winograd's Matrix Multiplications** 01-Jul-2008
(21 pages).

Kolda, Tamara G.; Bader, Brett W. [Sandia National Laboratories]: **Tensor Decompositions and Applications** 10-Jun-2008
(47 pages).

Kleitman, Daniel; Shor, Peter: **Principles of Applied Mathematics: Strassen's Fast Multiplication of Matrices Algorithm and Spreadsheet Matrix Multiplications** 09-May-2008
(11 pages).

Srivastava, Piyush: **Approximative techniques for embedding Matrix Multiplication in Group Algebras** 17-Apr-2008
In this article, we discuss a scheme of embedding matrix multiplication into a group algebra based on the concept of degeneration, and present conjectures which if true would lead to non-trivial estimates of the exponent of matrix multiplication using these schemes. (10 pages).

Bard, Gregory V. [Fordham University]: **On the Rapid Solution of Systems of Polynomial Equations over Low-Degree Extension Fields of GF(2), via SAT-Solvers.** 13-Mar-2008
(25 pages).

Landsberg, J.M.: **Geometry and the Complexity of Matrix Multiplication** 07-Jan-2008
We survey results in algebraic complexity theory, focusing on matrix multiplication. Our goals are (i) to show how open questions in algebraic complexity theory are naturally posed as questions in geometry and representation theory, (ii) to motivate researchers to work on these questions, and (iii) to point out relations with more general problems in geometry. The key geometric objects for our study are the secant varieties of Segre varieties. We explain how these varieties are also useful for algebraic statistics, the study of phylogenetic invariants, and quantum computing. (38 pages).

Gates, Ann Q.; Kreiniovich, Vladik [University of Texas at El Paso]: **Strassen's Algorithm Made (Somewhat) More Natural - A Pedagogical Remark** 07-Nov-2007
(4 pages).

Kolda, Tamara G.; Bader, Brett W. [Sandia National Laboratories]: **Tensor Decompositions and Applications** 01-Nov-2007
(71 pages).

Bini, Dario A. [Pisa University]: **The role of tensor rank in the complexity analysis of bilinear forms** 16-Jul-2007
(46 slides).

Landsberg, J.M.: **Geometry and the Complexity of Matrix Multiplication** 12-Mar-2007
We survey results in algebraic complexity theory, focusing on matrix multiplication. Our goals are (i.) to show how open questions in algebraic complexity theory are naturally posed as questions in geometry and representation theory, (ii.) to motivate researchers to work on these questions, and (iii.) to point out relations with more general problems in geometry. The key geometric objects for our study are the secant varieties of Segre varieties. We explain how these varieties are also useful for algebraic statistics, the study of phylogenetic invariants, and quantum computing. (34 pages).

Bartsch, Benjamin A.: **Laderman's algorithm for 3x3 matrix multiplication** 01-Jan-2007
Newsgroup log (7 pages).

Demmel, James; Dumitriu, Ioana; Holtz, Olga; Kleinberg, Robert [University of California at Berkeley]: **Fast matrix multiplication is stable** 07-Dec-2006
(19 pages).

Sikorski, K.; Boonyasiriwat, C.; Litchfield, K.; Xiong, C. [University of Utah]: **Selected Nonlinear Problems** 06-Sep-2006

Includes section 3.: Divide-and-conquer algorithms for matrix multiplication (40 slides).

Bard, Gregory V. [University of Maryland]: **Achieving a log(n) Speed Up for Boolean Matrix Operations and Calculating the Complexity of the Dense Linear Algebra step of Algebraic Stream Cipher Attacks and of Integer Factorization Methods** 05-May-2006
(20 pages).

Landsberg, J.M.: **The Border Rank of the Multiplication of 2 x2 Matrices is seven** 05-Apr-2006
(16 pages).

Brickell, Justin: **Practical Fast Matrix Multiplication** 25-Dec-2005
Project report (15 pages).

Bard, Gregory V.: **Algorithms for Fast Matrix Operations** 07-Dec-2005
(13 pages).

Cohn, Henry; Kleinberg, Robert; Szegedy, Balázs; Umans, Christopher: **Group-theoretic Algorithms for Matrix Multiplication** 10-Sep-2005
(12 pages).

Lang, Stefan [Munich University]: **Matrixalgebra mit einer Einführung in lineare Modelle** 14-Apr-2005
(213 pages). In German

Wayne, Kevin [Pearson-Addison Wesley]: **How to Multiply** 01-Jan-2005
Slides inspired by Jon Kleinberg - Éva Tardos: "Algorithm Design". (16 slides).

Lang, Stefan [Munich University]: **Matrixalgebra** 25-Aug-2004
(213 pages).

Yuster, Raphael; Zwick, Uri: **Fast sparse matrix multiplication** 21-Jun-2004
(11 pages).

Kaporin, Igor [Russian Academy of Sciences, Moscow]: **The aggregation and cancellation techniques as a practical tool for faster matrix multiplication** 01-May-2004
(42 pages).

Kakaradov, Boyko [Stanford University]: **Ultra-Fast Matrix Multiplication - An Empirical Analysis of Highly Optimized Vector Algorithms** 01-Apr-2004
(4 pages).

Widgerson, Avi [Princeton University]: **Arithmetic Complexity - A Survey** 23-Dec-2003
(13 pages).

Kleitman, Daniel; Shor, Peter: **Principles of Applied Mathematics: Strassen's Fast Multiplication of Matrices Algorithm and Spreadsheet Matrix Multiplications** 23-Dec-2003
(9 pages).

Cohn, Henry; Umans, Christopher: **A Group-theoretic Approach to Fast Matrix Multiplication** 31-Oct-2003
(12 pages).

Zerbe, Volker [Technische Universität Ilmenau]: **Parallele Matrizenmultiplikation** 26-Jun-2003
(129 pages).

Pan, Victor Y. [City University of New York]: **Randomized Acceleration of Fundamental Matrix Computations** 23-Dec-2002
(13 pages).

Meppelink, David J.; Canning, James [University of Massachusetts Lowell]: **Matrix Multiplication Performance** 17-Dec-2002
(12 pages).

Raz, Ran [Weizmann Institute]: **On the Complexity of Matrix Product** 01-Dec-2002
(18 pages).

Pippenger, Nicholas: **Algebraic Complexity Theory** 17-Sep-2002
Algebraic complexity theory, the study of the minimum number of operations suficient to perform algebraic computations, is surveyed with emphasis on the general theory of bilinear forms and two of its applications: polynomial multiplication and matrix multiplication. Though by no means exhausting algebraic complexity theory, these topics illustrate well its development and its methods, and provide examples of its most striking successes. (8 pages).

Bläser, Markus [Lübeck University]: **On the complexity of the multiplication of matrices of small formats** 10-Jun-2002
We prove a lower bound of 2mn + 2n - m - 2 for the bilinear complexity of the multiplication of n x m-matrices with m x n-matrices using the substitution method (m >= n >

Alon, Noga; Galil, Zvi; Margalit, Oded; Naor, Moni [Tel Aviv University]: **Witnesses for Boolean Matrix Multiplication and for Shortest Paths** 20-Feb-2002
(10 pages).

Kaporin, Igor: **The Aggregation and Cancellation Techniques As a Practical Tool for Faster Matrix** 01-Jan-2002

Multiplication
(38 pages).

Cormen, Thomas H.; Leiserson, Charles E.; Rivest, Ronald L.; Stein, Clifford: **Intrduction to Algorithms: Strassen's algorithm for matrix multiplication** 01-Sep-2001
Chapter 28.2, page 642, (5 pages).

Kolen, John F.; Bruce, Phillip Bruce [University of West Florida]: **Evolutionary Search for Matrix Multiplication Algorithms** 25-Jul-2001
This paper addresses the problem of algorithm discovery, via evolutionary search, in the context of matrix multiplication. The traditional multiplication algorithm requires $O(n^3)$ multiplications for square matrices of order n. Strassen (Strassen 1969) discovered a recursive matrix multiplication algorithm requiring only seven multiplications at each level, resulting in a runtime of O(n^lb7), or O(n^2.81). We have been able to replicate this discovery using evolutionary search (Fogel 1995). The paper presents the representational schema, evaluation criteria, and evolution mechanisms employed during search. The most crucial decision was removing the determination of coefficients used to combine the product terms in the final addition steps from the search space and calculating them directly from the specified multiplications. Extending this methodology from 2 x 2 submatrices to algorithms using 3 × 3 decompositions is also discussed. (5 pages).

Aberdeen, Douglas; Baxter, Jonathan [Canberra University]: **Emmerald: a fast matrix-matrix multiply using Intel's SSE instructions** 08-Feb-2001
(17 pages).

Bürgisser, Peter [Universität Zürich]: **Algebraische Komplexitätstheorie II: Schnelle Matrixmultiplikation und Kombinatorik** 07-Aug-2000
(16 pages).

Yagle, Andrew E. [University of Michigan]: **Fast Matrix Computations** 02-Jul-2000
(10 pages).

Kaporin, Igor [Russian Academy of Sciences, Moscow]: **A Practical Algorithm for Faster Matrix Multiplication** 25-Nov-1999
(14 pages).

Siek, Jeremy G. [University of Notre Dame]: **A Modern Framework for Portable High Performance Numerical Linear Algebra** 01-Apr-1999
This thesis describes a generic programming methodology for expressing data struc- tures, algorithms, and optimizations for numerical linear algebra. A high-performance implementation of this approach, the Matrix Template Library (MTL), is also described. The goal of the MTL is to facilitate development of higher-level libraries and applications for scientific computing. In addition, the programming techniques developed in this thesis are widely applicable and can be used to reduce development costs, improve readability, and improve the performance of many kinds of software. Portable high performance is a particular focus of the MTL. Flexible kernels were constructed that provide an automated tool for cross architecture performance portability. (297 pages).

Xiongda, Chen [Chinese Academy of Sciences]: **Implementation of Hopcroft-Kerr Algorithm on SR2201** 01-Jan-1999
(9 pages).

Thottethodi, Mithuna; Chatterjee, Siddhartha; Lebeck, Alvin R.: **Tuning Strassen's Matrix Multiplication for Memory Efficiency** 08-Jun-1998
trassen's algorithm for matrix multiplication gains its lower arithmetic complexity at the expense of reduced locality of reference, which makes it challenging to implement the algorithm efficiently on a modern machine with a hierarchical memory system. We report on an implementation of this algorithm that uses several unconventional techniques to make the algorithm memory-friendly. First, the algorithm internally uses a non-standard array layout known as Morton order that is based on a quad-tree decomposition of the matrix. Second, we dynamically select the recursion truncation point to minimize padding without affecting the performance of the algorithm, which we can do by virtue of the cache behavior of the Morton ordering. Each technique is critical for performance, and their combination as done in our code multiplies their effectiveness. Performance comparisons of our implementation with that of competing implementations show that our implementation often outperforms the alternative techniques (up to 25). However, we also observe wide variability across platforms and across matrix sizes, indicating that at this time, no single implementation is a clear choice for all platforms or matrix sizes. We also note that the time required to convert matrices to/from Morton order is a noticeable amount of execution time (515175760 15). Eliminating this overhead further reduces our execution time.

Livingston: **Efficient Matrix Multiplication** 01-Jan-1998
Research proposal In previous work R. Johnson and A. McLoughlin, by a computer-aided search, found new noncommutative bilinear algorithms for 3×3 matrix multiplication that require only 23 essential multiplications rather than the 27 required by the conventional method. Such algorithms, like Strassen's algorithm for the 2×2 case, lead to fast algorithms for matrices of arbitrary size. It is proposed to extend the search to obtain improved upper bounds on the number of essential multiplications required for the 3×3 case and other small sizes, in particular 4×4. Implications of the results for the design of special-purpose array-processing hardware will be studied. (3 pages).

Huang, Xiaohan; Pan, Victor Y. [City University of New York]: **Fast Rectangular Matrix Multiplication and** 21-Jan-1997

Applications
(43 pages).

Huss-Lederman, Steven; Jacobson, Elaine M.; Johnson, J.R.; Tsao, Anna; Turnbull, Thomas: Strassen's Algorithm for Matrix Multiplication:Modeling, Analysis, and Implementation — 15-Nov-1996
In this paper we report on the development of an efficient and portable implementation of Strassen's matrix multiplication algorithm for matrices of arbitrary size. Our implementation is designed to be used in place of DGEMM, the Level 3 BLAS matrix multiplication routine. Our code is designed so that efficient performance will be obtained for all matrix sizes and shapes and that the additional memory needed for temporary variables is minimized. Replacing DGEMM with our routine should provide significant performance gain for large matrices while providing the same performance for small matrices. We measure performance of our code on the IBM RS/6000, CRAY YMP C90, and CRAY T3D single processor, and offer comparisons to other codes. Our performance data reconfirms that Strassen's algorithm is practical for realistic size matrices. The usefulness of our implementation is demonstrated by replacing DGEMM with our routine in a large application code. (70 pages).

Huss-Lederman, Steven; Jacobson, Elaine M.; Johnson, Jeremy R.; Tsao, Anna; Turnbull, Thomas: Implementation of Strassen's Algorithm for Matrix Multiplication — 01-Aug-1996
(27 pages).

Bürgisser, Peter [Universität Zürich]: Schnelle Matrixmultiplikation und Kombinatorik — 01-Jan-1996
In 1969 Strassen discovered that Gaussian elimination is not an optimal algorithm for solving various problems in computational linear algebra. His result was based on a fast matrix multiplication algorithm needing only $O(n\tau)$ arithmetic operations, where $\tau < 2.81$. The infimum of all possible exponents $\tau >< 2.38$. Today, one even conjectures that $ømega = 2$. We survey the main ideas and methods, which have led to such insights about the complexity o

Grayson, Brian; Shah, Ajay Pankaj; van, Robert A. [University of Texas at Austin]: A High Performance Parallel Strassen Implementation — 13-Jun-1995
In this paper, we give what we believe to be the first high performance parallel implementation of Strassen's algorithm for matrix multiplication. We show how under restricted conditions, this algorithm can be implemented plug compatible with standard parallel matrix multiplication algorithms. Results obtained on a large Intel Paragon system show a 10-20reduction in execution time compared to what we believe to be the fastest standard parallel matrix multiplication implementation available at this time. (10 pages).

Knoght, Philip A. [University of Strathclyde]: Fast Rectangular Matrix Multiplication and QR Decomposition — 01-May-1995
(13 pages).

Bini, Dario; Pan, Victor Y. [Birkhäuser]: **Polynomial and matrix computations. Fundamental algorithms. Vol.1** — 01-Jan-1994
(416 pages).

Bini, Dario; Pan, Victor Y. [Birkhäuser]: **Polynomial and matrix computations. Fundamental algorithms. Vol.1** — 01-Jan-1994
(416 pages).

Anderson, N. Anderson; Manley, D. Manley [Digital Equipment Coproration]: A matrix extension of Winograd's inner product algorithm — 01-Feb-1993
(3 pages).

Strassen, Volker [Universität Konstanz]: Algebra and Complexity — 01-Jul-1992
(16 pages).

Julian Laderman, Victo Y. Pan; Sha, Xuan-He: On Practical Algorithms for Accelerated Matrix Multiplication — 01-Feb-1992
We present several bilinear algorithms for the acceleration of multiplication of n X n matrices that are superior to both the classical and Strassen's algorithm for moderate n (starting with n = 20). The Bini-Lotti result on the weak stability of bilinear algorithms applies to these algorithms. We present them in two equivalent versions, bilinear and trilinear. We also apply one of these algorithms over finite fields (or rings) of constants. Surprisingly, this enables us to decrease the bilinear complexity of n X n matrix multiplication (for 20 <= n <

Zhang, Xing [Western Carolina University]: Choosing a Better Algorithm for Matrix Multiplication — 01-Jan-1991
This thesis conducts the study of three algorithms; the straightforward algorithm, Winograd's algorithm, Strassen's algorithm, their time complexities, and compares the three algorithms using graphs. The thesis also briefly describes two asymptotic improvements: Pan's of 1983 and Strassen's of 1986. (91 pages).

Huang, C.H.; Johnson, J.R.; Johnson, R.W.: A tensor product formulation of Strassen's matrix multiplication algorithm — 01-Feb-1990
Appl. Math. Lett. Vol. 3, No. 3, pp. 67-71, 1990 (5 pages).

Huang, C.-H.; Johnson, J.R.; Johnson, R.W.: A Tensor Product Formulation of Strassen's Matrix Multiplication Algorithm — 01-Feb-1990
(5 pages).

Bshouty, Nader H. [Technion - Israel Institute of Technology]: A lower bound for matrix multiplication — 03-Nov-1988
(11 pages).

Bailey, David H. [NASA]: **Extra-High Speed Matrix Multiplication on the Cray-2** 01-May-1988
(7 pages).

Coppersmith, Don; Winograd, Shmuel [IBM]: **Matrix Multiplication via Arithmetic Progressions** 17-May-1987
(31 pages).

Makarov, O. M.: **Makarov-1987-A_non-commutative_algorithm_for_multiplying_5_x_5_matrices_using_one_hundred_multiplications** 01-Jan-1987
(3 pages). translated version

Johnson, Rodney W.; McLoughlin, Aileen M.: **Noncommutative Bilinear Algorithms for 3x3 Matrix Multiplication** 01-May-1986
New noncommutative bilinear algorithms for 3 x 3 matrix multiplication are presented. These have the same complexity, 23 essential multiplications, as the one discovered by Laderman, but are inequivalent to it. Equivalence here refers to a certain group of transformations all of which map noncommutative bilinear matrix-multiplication algorithms into other such algorithms; "inequivalent" means not related by a transformation in the group. This group has been studied by de Groote, who has shown for the case of 2 x 2 matrix multiplication with 7 essential multiplications that all such algorithms are equivalent to Strassen's. The new algorithms, by contrast, include infinitely many pairwise inequivalent algorithms. The computer search that led to the new algorithms is described. (9 pages).

Makarov, O. M.: **An algorithm for multiplying 3x3 matrices** 01-Jan-1986
(2 pages). in Russian language!

Pan, Victor Y.: **How can we speed up Matrix Multiplication?** 01-Jul-1984
(24 pages).

Coppersmith, Don: **Rapid Multiplication of Rectangular Matrices** 01-Aug-1982
(5 pages).

Alekseyev, Valery B. [Moscow State University]: **On the Complexity of Some Algorithms of Matrix Multiplication** 05-May-1982
(15 pages).

Schönhage, Arnold [Tübingen University]: **Partial and Total Matrix Multiplication** 01-Aug-1981
(22 pages).

Bini, Dario A. [Pisa University]: **Relations between exact and approximate bilinear Algorithms** 10-Apr-1979
(11 pages).

Brockettt, Roger W.; Dobkin, David [Harvard University]: **On the Optimal Evaluation of a Set of Bilinear Forms** 01-Jan-1978
(29 pages).

Sýkora, Ondrej [Slovak Academy of Sciences]: **A Fast Non-Commutative Algorithm for Matrix Multiplication** 01-Jan-1977
In the paper a non-commutative algorithm for the multiplication of two square matrices of order n is presented. The algorithm requires n3-(n-1)2 multiplications and n3-n2+ 11 (n-1)2 additions. The recursive application of the algorithm for matrices of order nk leads to O(klogn[n3-(n-1)2]n) operations to be executed.It is shown that some well-known algorithms are special cases of our algorithm. Finally, an improvement of the algorithm is given for matrices of order 5. (6 pages).

Laderman, Julian D.: **A Noncommutative Algorithm for Multiplying 3 x 3 Matrices Using 23 Multiplications** 09-Oct-1975
(3 pages).

Makarov, O. M.: **Makarov - 1975 - The connection between the fast transform algorithms of Fourier and Hadamard and the algorithms of Karacuba, Strassen and Winograd** 01-Jan-1975
(11 pages). in Russian language! English version:
http://www.sciencedirect.com/science/article/pii/0041555375900993

Hopcroft, John E.; Musinski, J. [Cornell University]: **Duality Applied to the Complexity of Matrix Multiplications and other Bilinear Forms** 01-Jan-1973
The paper considers the complexity of bilinear forms in a noncommutative ring. The dual of a computation is defined and applied to matrix multiplication and other bilinear forms. It is shown that the dual of an optimal computation gives an optimal computation for a dual problem. An nxm by mxp matrix product is shown to be the dual of an nxp by pxm or an mxn by nxp matrix product implying that each of the matrix products requires the same number of multiplications to compute. Finally an algorithm for computing a single bilinear form over a noncommutative ring with a minimum number of multiplications is derived by considering a dual problem. (15 pages).

Probert, Robert L.: **On the Complexity of Symmetric Computations** 01-Jan-1973
(31 pages).

Brent, R. P. [Stanford University]: **Algorithms for Matrix Multiplication** 01-Mar-1970
Strassen's and Winograd's algorithms for n × n matrix multiplication are investigated and compared with the normal algorithm. The normal algorithm requires n3 + O(n2) multiplications and about the same number of additions.

Winograd's algorithm almost halves the number of multiplications at the expense of more additions. Strassen's algorithm reduces the total number of operations to O(n2.82) by recursively multiplying 2n × 2n matrices using seven n × n matrix multiplications. Floating-point error bounds are obtained for both Strassen's and Winograd's methods. It is shown that Strassen's method satisfies a certain numerical stability property (albeit weaker than that satisfied by the normal method); and that scaling is essential for numerical accuracy using Winograd's method. In practical cases, for moderate n, Winograd's method appears to be slightly faster than the other two methods, but the gain is, at most, about 20 percent. Strassen's method should be faster for sufficiently large n, and this will be important in the future as memory sizes increase. An attempt to generalize Strassen's method is described. (55 pages).

Hopcroft, John .E.; Kerr, Leslie Robert [Cornell University]: **On Minimizing the Number of Multiplications Necessary for Matrix Multiplication** — 01-Sep-1969

This paper develops an algorithm to multiply a px2 matrix by a 2xn matrix in $\lceil (3pn+max(n,p))/2 \rceil$ multiplications for matrix multiplication without commutativity. The algorithm minimizes the number of multiplications for matrix multiplication without commutativity for the special cases p=1 or 2, n=1,2, $\cdots$ and p = 3, n = 3. It is shown that with commutativity fewer multiplications are required. (34 pages).

Waksman, Abraham [Stanford Research Institute]: **On Winograd's Algorithm for Inner Products** — 04-Aug-1969

(2 pages).

Strassen, Volker [Zurich University]: **Gaussian Elimination is not optimal** — 12-Dec-1968

(3 pages).

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